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A COMPARISON OF THE BASKET METHOD AND STRATIFIED RANDOM SAMPLING FOR CONTRACT CHANGE ORDER NEGOTIATIONS

by

Scott W. Fisher

June 1987

Thesis Advisor: Shu S. Liao

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A Comparison of The Basket Method and Stratified Random Sampling for Contract Change Order Negotiations

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ABSTRACT

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TABLE OF CONTENTS

I. INTRODUCTION 7
  A. PURPOSE OF THIS STUDY 7
  B. NEED FOR THIS STUDY 7
  C. METHODOLOGY 9

II. THE BASKET METHOD 11
  A. HISTORY 11
  B. DESCRIPTION 11
    1. Basket Assignment 11
    2. Estimating Negotiated Prices for Unsampled Proposals 12

III. STRATIFIED RANDOM SAMPLING 13
  A. THE CASE FOR STRATIFICATION 13
  B. DESCRIPTION OF STRATIFIED RANDOM SAMPLING 14
    1. Establish the Desired Precision and Reliability 14
    2. Designate the Strata and Strata Boundaries 15
    3. Sample Size Determination and Allocation 17
    4. Select a Random Sample of Size n_i From the Strata 19
    5. Calculate the Mean of Each Stratum Based On n_i for each Stratum . . . 19
    6. Calculate the Estimated Audited Population Total 19
    7. Check Reliability of the Estimated Audited Population Total . . . 19

IV. DESCRIPTION OF SIMULATION 20
  A. DERIVING COMPARABLE RESULTS 20
  B. CREATION OF THE TEST POPULATIONS 20
  C. SIMULATION EXECUTION 21
    1. Stratify the Population 22

    2. Allocate the Total Sample to the Strata 22
    3. Select a Random Sample From Each Strata 22
    4. Calculate the "Book Value" sum for Audited Items 22
    5. Calculate the "Audit Value" sum for Audited Items 22
    6. Calculate the Correction Factor 22
    7. Calculate the Predicted Population Audit Total 22
    8. Calculate the Percent Error 22

V. SUMMARY AND CONCLUSIONS 26
  A. RESISTANCE TO PROPOSAL RIGGING 26
  B. EVALUATION 26
  C. AREAS FOR FURTHER RESEARCH 27

APPENDIX A: BASKET METHOD PROGRAM LISTING 28

APPENDIX B: MINITAB SIMULATION OF STRATIFIED RANDOM SAMPLING 31

APPENDIX C: DETAILED RESULTS 35

LIST OF REFERENCES 39

INITIAL DISTRIBUTION LIST 40

LIST OF TABLES

1. POPULATION DESCRIPTION 21
2. SIMULATION RESULTS (MEAN % ERROR) 23
3. BASKET METHOD ERROR AS A % OF STRATIFIED RANDOM SAMPLING ERROR 24
4. STANDARD DEVIATION OF MEAN PERCENT ERROR 25

I. INTRODUCTION

A. PURPOSE OF THIS STUDY

The purpose of this study is to examine two methods of statistical sampling which may have application in assisting government price analysts and contract negotiators in expediting processing of proposals for change orders. The study will describe how and why the use of statistical sampling may expedite proposal negotiation while maintaining acceptable levels of risk, and will determine which of the two sampling methods examined provides more acceptable results under the given conditions.

B. NEED FOR THIS STUDY

Current defense acquisition procedures often involve situations in which Department of Defense (DOD) agencies must deal with a sole source supplier in buying material or services. In major weapon system acquisitions for example, the Department of Defense typically issues a large number of change orders to modify an existing contract. A lead ship or aircraft production contract may generate over 10,000 change orders. Why must such a large volume of change orders be issued? After a prime contract is awarded and production begins, design changes are often necessitated by a change in performance requirements requested by the government or by unforseen technical problems which almost always seem to crop up. Each design change requires a modification to the prime contract called a change order. In each case, the contractor prepares a proposal reflecting his estimate of what the requested change will cost. The two parties (government and contractor) must then negotiate a price for each change; and, because the prime contractor is the logical one to incorporate the requested change, there is no competition to help assure that the government receives the fairest possible price. The only mechanisms working to assure a fair price for the change order are the adequacy of the contractor's estimating procedures, the contractor's inherent honesty and desire to provide a good product at a fair price, and the analysis of the proposal by government price analysts.

Federal Acquisition Regulations (FAR) require the government to analyze each proposal prior to negotiation to assure that the proposal represents a fair price. The analysis and negotiation of costs for each proposal is done by some cognizant government agency. Often, a group of government employees have been assigned to perform such functions in residence at the contractor's plant. The volume of work thus generated and the amount of money involved are quite substantial. This volume of work combined with a lack of sufficient numbers of government analysts leads to large backlogs of unprocessed proposals. To perform a really thorough analysis and patient negotiation takes much more time than government analysts are currently able to give to a proposal. If analysts do try to take more time and be more thorough, they fall still farther behind as the backlog continues to grow. Therefore, there is tremendous pressure on analysts to expedite their work even though it is generally recognized that hurried analysis and negotiation can result in costly overpayment since quickness commonly works against thoroughness and accuracy [Ref. 1: pg. 2].

Unprocessed proposals can result in extra expense for the contractor too. In many situations involving ongoing production or repair work, the proposed work is begun before the proposal is analyzed and negotiated to avoid expensive delay and disruption costs. The contractor, however, except for partial advances called "progress payments", is not paid until after the proposal is processed. Because of this, the contractor may have to borrow working capital to cover funds tied up in the backlog thus suffering capital costs.

It is generally recognized that it is in the best interest of both parties to expedite the processing of the proposals without sacrificing accuracy. If allowed by the regulations, analyzing and negotiating a sample of proposals selected with an effective statistically-based sampling technique could have these effects. If a suitable sample of proposals were selected from the backlog and carefully analyzed and negotiated, the resulting data could be extrapolated to estimate what the results would have been had every proposal received the same treatment.

The reason for auditing the proposal population is to ensure the proposals reflect costs that are fair and reasonable. During an audit the government analyst will find that one of three possible conditions exists. First, the government may feel that the proposal has been understated, i.e., the contractor's proposed cost for a change to the contract is less than the actual cost the contractor will incur. Second, the government may conclude that the contractor's proposal is overstated. Third, the audit findings may conclude that the proposal is reasonable. Any overstatement or understatement is considered to be an error in the proposal population. A sampling technique which allowed no sampling error (with sampling error being defined as the chance that a sample which is statistically selected and evaluated will lead to the wrong conclusion or to an inaccurate projection) would select a sample from the population of proposals which, when audited, would always give an estimated value for the population as a whole which was exactly correct, no matter what the degree or distribution of errors in the proposal population. Since sampling errors are due entirely to chance and are inherent in any sampling process, we cannot expect a sample of "n" proposals from the proposal population to provide an error-free characterization of the "N" proposals in the population.

Assuming, then, that there will be some degree of error in the prediction of the true value of the entire proposal population whenever a sampling technique is used; the behavior of the degree of error must be predictable and exhibit certain qualities in order for the sampling technique to be considered appropriate for the purpose described above.

Specifically, the degree of error should not be easily altered by the distribution, size, or type (overstatement/understatement) of errors found in the population. If certain patterns of errors caused the entire population to be evaluated as understated then the government would pay more than a fair price for the changes described by the population of proposals which contained those errors [Ref. 2: pg. 7]. Since both parties to the negotiation would have to enter into a binding agreement to abide by the results of using statistical methods, all aspects of the sampling and estimation process must be disclosed in advance. With this necessary advanced knowledge, a shrewd contractor could carefully seed his proposal population with deliberate errors of the appropriate size, type, and distribution and thereby be awarded a larger payment from the government.

Therefore, the desired sampling technique will not necessarily be the one which results in the most accurate, average estimate for various error arrangements in the proposal population. It will instead be the method which responds least to variations in the arrangement of errors.

C. METHODOLOGY

As mentioned previously, the purpose of this study is to examine the effectiveness of two sampling techniques that appear to be most suitable for the purpose of analyzing contract change orders. The two sampling techniques to be studied are Stratified Random Sampling and the Basket Method. In this study, the two methods will be used to draw samples from populations for evaluation. The populations were previously used in a joint study of the American Institute of Certified Public Accountants and the American Statistical Association [Ref. 3]. The data consist of two columns of values which represent the proposed, or book value of a contract change and the audited or true value of the change. The populations are rigged with either random or planned errors. The samples drawn by the two methods from each population will be evaluated and compared to determine which method gives a better estimate of the whole population according to the goals described above. Both the error rigging and evaluation steps are explained further in the description of the simulation.

The amount of work associated with auditing is more closely correlated to the number of items being audited than to the total dollar value of all the items being audited. Therefore, the sampling rules of the two methods will be adjusted so that they will draw samples with the same number of proposals from each population. The results of this study will then indicate which method yields the more desirable prediction while holding the cost of the audit constant.

II. THE BASKET METHOD

A. HISTORY

The "Basket Method" of sample selection was developed by Dr. K. T. Wallenius, Professor of Mathematical Sciences at Clemson University. Development of the Basket Sampling method was sponsored by the Office of Naval Research and Naval Material Command and funded by the Office of Naval Research under its Acquisition Research program. The Basket Method was developed as a potential tool to assist price analysts and contract negotiators in expediting processing of proposals for change orders when dealing with a sole source supplier.

B. DESCRIPTION

The name "Basket Method" is derived from the manner in which the population is partitioned into separate groups (baskets) prior to randomly selecting one of the baskets as the sample. The goal of partitioning the population into baskets by the basket assignment process is to make each basket a good representation of the population as a whole. It must be stressed at this point that "representative" should be thought of in terms of ***bid prices*** only.[1] Because each basket is representative of the population as a whole, the spread and proportion of proposal values will be nearly identical to those of the population. Therefore, it makes no difference which basket is selected to be audited in detail. The following example will describe the use of the basket method technique.

1. Basket Assignment

Imagine having a population of 100 proposals ($N = 100$) from which a 10% sample ($n = 10$) is to be selected. The proposals are then arranged in order of decreasing bid price and numbered accordingly; that is, the proposal with the largest bid price is number 1, the second largest number 2, and so on. The proposals are now ready to be separated into 10 different baskets. Starting with proposals 1 through 10 (those with the largest bid prices), one proposal is placed in each basket. Each successive group of 10 proposals are assigned, one-per-basket, using the following rule: *the largest unassigned proposal is placed in the basket with the smallest sum of bid prices.* For the second group of 10 proposals, this rule results in pairing proposal 11 with 10, 12 with 9, . . ., and 20 with 1. Basket subtotals are then calculated and the assignment rule applied to the third group of 10 proposals. This is repeated until all the proposals have been assigned. [Ref. 1: pg.10]

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[1] It is realized there may be other relevant factors besides bid price that should be considered in the definition of "representative". For the purposes of this paper, however, it will suffice to say that sophisticated software can quickly balance baskets for type of work, degree of labor intensity, level of technology, etc. In short, whatever characteristics are identified as potentially important to the value of an audit will be "balanced" by the basket method where possible.

Due to the balancing of basket totals at each stage of the basket assignment process, the resulting assignment should result in nearly equal basket totals. Should additional balancing be required, the previously mentioned computer program can be used (via a swapping algorithm) to bring basket totals into closer agreement.

2. Estimating Negotiated Prices for Unsampled Proposals

After the baskets are formed, one is selected at random and all its proposals are audited and negotiated. Using the results of the sample negotiation, the sample ratio factor, R, is computed as in equation 2.1.

$$R = \text{Total negotiated price of sample}/\text{Total bid price of sample} \qquad \text{(eqn 2.1)}$$

The total proposal value of the population is then multiplied by the sample ratio factor, R, to determine the population audit result. This value will be the *estimated true value* of the population.[2]

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[2] The sample ratio factor could also be applied individually to each unsampled proposal, the values summed and the total added to the sum of the negotiated values of sampled proposals. The result would be the same.

III. STRATIFIED RANDOM SAMPLING

A. THE CASE FOR STRATIFICATION

Stratified random sampling is similar in many respects to the technique of unrestricted random sampling.[3] The major difference is that the population is divided into two or more groups (strata), each of which is then sampled separately. The results can then be combined to give an estimate of the total population value.

The primary objective of stratification in auditing is to reduce the impact of the population variance on the sampling plan. Basically, a population of heterogeneous items (a population with large variance) is broken into two or more groups or strata of a more homogeneous nature (groups with small variances). The total population variance is unaffected by this process. However, it should be intuitively clear that within each group so constructed, the strata variance will be smaller than the population variance. [Ref. 4: pg. 149]

To illustrate, suppose a population consists of seven items--five have a value of $1 each, and two have a value of $3 each. The variance of this population is close to $1, but by forming two strata with the five items valued at $1 each in one stratum and the remaining two items of value $3 in the other stratum, the variation of each stratum is 0. This reduction in variance by the formation of two strata has important implications for the amount of sampling error and the size of the sample required. The relationship can be summarized as follows: Given any population of size N, the lower the variability, the smaller the sample size required to achieve any given precision[4] and reliability[5] requirements. [Ref. 5: pg. 12].

While the above example is very simplistic and hypothetical in nature, it does illustrate the fact that by taking a relatively heterogeneous population and dividing it up into homogeneous groups the variance of each group will be smaller than that of the original population. As a result, the sample size required will be smaller than if unrestricted random samples were taken; or alternatively, the reliability would be higher or the precision limits narrower. Stratification should therefore be applied to heterogeneous populations which can be divided into fairly uniform strata on the basis of some criteria that affects the variable being studied. Under these circumstances, stratification usually achieves greater precision for a given cost. On the other hand, stratification is unnecessary in homogeneous populations where there are no discernible strata that will affect the results.

---

[3] The principle involved in unrestricted random sampling is that every element in the population should have an equal chance of being included in the sample. Since "randomness" is difficult to achieve without some kind of aid, a random number table or a computerized random number generator are often used to insure random selection.

[4] The range within which the true answer most likely falls.

[5] The likelihood that the true answer will fall within the established range. It is usually expressed as a percentage, being the number of times out of one hundred that the true answer would be contained within the determined margins.

To use stratified sampling, three general rules must be adhered to [Ref. 6: pg. 96]:

1. Every element must belong to one and only one stratum.
2. There must be a tangible, specifiable difference that defines and distinguishes the strata.
3. The exact number of elements in each stratum must be known.

B. DESCRIPTION OF STRATIFIED RANDOM SAMPLING

Once the decision has been made that stratification would be beneficial in the sampling process, there are several steps that must be taken. These steps will be briefly discussed below.

1. Establish the Desired Precision and Reliability

Statistical samples are evaluated in terms of "precision," which is expressed as a range of values, plus or minus, around the sample result, and "reliability" (or confidence), which is expressed as the proportion of such ranges from all possible similar samples of the same size that would include the actual population value. [Ref. 7: pg. 4]

Basically, the statistical measures of precision and reliability have to do with how accurate and reliable the sampler wants his sample results to be. An example of the application of these two measures is helpful in understanding the concepts. Suppose an auditor is designing a statistical test based on a desire to obtain an estimate of an audited account value to within $10,000. The $10,000 amount reflects the auditor's judjment as to what would constitute a material deviation in reported values. In other words, the auditor does not want his estimate of the audited account value to be greater than $10,000 (either plus or minus) away from the true audited account value. Reliability is a closely related concept. The auditor's goal is not only to obtain an estimate within the materiality limit of $10,000 but also to be reasonably sure that this estimate is sound. Because only a sample is observed judgmentally or statistically in most audit situations, certainty is impossible. Generally accepted auditing standards recognize this by requiring reasonable assurance rather than certainty. Reliability is the statistical measure of that level of assurance stated as a proportion. For example, a proportion of 0.95 indicates that the auditor wishes to achieve a 95% level of reliability that the reported amount is not materially different (plus or minus $10,000) from the audited amount.

Specification of a probable range for a population parameter--a plus or minus for error--is crucial in indicating the reliability of estimates. This process involves the construction of a confidence interval for the population parameter being estimated. An in-depth look at confidence level construction is beyond the scope of this study however, and it is suggested that the reader consult any good statistics textbook for a detailed discussion of this topic. For this study, no specific precision and reliability levels will be set; the purpose of this study being to compare the results of the two sampling methods to each other rather than to attain some specific level of accuracy and reliability.

2. Designate the Strata and Strata Boundaries

For all practical purposes, there is currently no existing way to select the optimal number of strata or the strata boundaries [Ref. 4: pg. 158]. Useful rules do exist, however. Ideally, the auditor prefers to base stratification decisions on the specific variable of interest. In most audit applications, the variable of interest is the number of audited account values. There is a problem here, however, in that the number of audited account values is not actually known until after sampling. Fortunately, a good substitute for audited account values--reported account values(book values)--is usually available. The auditor generally expects a reasonably high correlation between the available reported account values and the obtained audit account values, and can be reasonably confident about basing stratification decisions on the available unaudited reported account values. However, this does limit the benefits of stratification in that, all other factors being equal, unless the correlation between reported and audited account values is perfect, the errors introduced by a particular audited account value belonging to different strata than the related reported account values will eventually negate any further benefits that can be obtained by the addition of new strata.

It is probably somewhat clear by now that, from a practical viewpoint, the identification of strata is a heuristic process (a sort of educated guess). In an auditing context, the approach that is most likely to be beneficial is to obtain some idea of the underlying character and distributional properties of the population of reported account values (book values). This can be done manually, but the results are much more meaningful when a computer can be utilized. The various output obtainable from a computer, along with a basic understanding of the data, may enable the auditor to subjectively select strata of a reasonable nature. In some cases, the data may lend themselves to obvious strata divisions, but in most situations this will probably not be the case.

Even if there are a certain number of obvious strata, say two, there are further questions to be asked. For example, if the use of two strata contribute to a substantial decline in the population variance, one might reasonably ask, "If two strata gave good results in reducing variance, wouldn't the use of four strata give results that are twice as good"? The answer is, although an increase to four strata might also be beneficial, it would probably not lead to as large a reduction in the variance estimate. In fact, such diminishing returns are observed as the number of strata increases. The first doubling of strata--from one to two--can produce variance reductions of as much as 60% or 70% [Ref. 4: pg. 159]. However, a second and third doubling tend to curtail the incremental reductions to about 25% [Ref. 4: pg. 159]. Therefore, there is some point at which the addition of more strata will no longer be useful in reducing variance estimates, and may in fact increase variance. The only practical way to establish the limits of strata benefit is by computer simulation. As a general rule, 5 to 10 strata usually account (depending on the particular population, of course) for most of the available variance reduction.

Given the number of strata, the auditor must then determine how and where to set strata boundaries. Ideally, strata boundaries should be established on the basis of audited account values, as before. But when these amounts are not available, reported (book) account values are commomly used as the basis for setting most boundary values. This substitution will work well if reported account values and audited account values are closely correlated.

Strata boundaries might be set using the *equal dollar value per strata rule*, which, as the name implies, means arranging the strata boundaries such that each strata has approximately the same dollar value; or, boundaries might be established based on the *equal variance rule* where each strata has approximately the same variance measure.

Another rule, sometimes referred to as the *Q-SUM* or *CUSUM rule*, establishes the strata boundaries by first creating a frequency distribution of the recorded (book) account values. The square root of the frequency of recorded account values in each category is then computed and summed and the resulting total is divided by the desired number of strata. The auditor attempts to create strata by accumulating the squared frequency measures in sequence until the cumulated sum (CUSUM) is approximately equal to the total accumulation divided by the number of strata. The next strata is then composed of the next grouping in the sequence such that the CUSUM is approximately equal to twice the total accumulation divided by the number of strata. [Ref. 4: pg. 161]

For this study, the population will be divided into 10 strata based on the book value amount of the audit unit. Stratification by book amount is helpful when the book amounts of the audit units are related to their audit values [Ref. 3: pg. 77]. The choice of 10 strata was made to facilitate comparison with the Basket Method in that 10 "baskets" will be used when applying the Basket Method in this study.

Strata boundaries will be set using the *equal dollar value per strata rule*. Again, this rule is used to facilitate comparison with the Basket Method where basket totals are nearly equal due to the unique basket assignment process. It may seem that if the "equal dollar value per strata" rule is used that, conceptually, there is no difference between the "strata" formed under Stratified Random Sampling and the "baskets" formed using the Basket Method. There is in fact a significant difference that stems from the distinctive ways in which the strata and baskets are formulated. Under the Basket Method, the population is partitioned into baskets in such a way that each basket will have approximately equal dollar value and contain approximately the same number of individual elements. Under the Stratified Random Sampling Method, strata are also partitioned so that they contain approximately equal dollar value but the number of individual elements in each strata may vary drastically.

3. Sample Size Determination and Allocation

Two methods are generally used to allocate a total sample to individual strata [Ref. 6: pg. 97]. One method is known as *proportional allocation*. In this method, the percentage of the sample allocated to each stratum is the same as the percentage of the total population accounted for by that stratum. That is,

$$n_i = n \times N_i/N \qquad \text{(eqn 3.1)}$$

where n_i represents the sample size for the ith stratum, n the total sample size, N_i the number of population items in the ith stratum, and N the total population size.

A generally more effective method, however, is *optimal allocation.* Optimal allocation allocates the total sample to the individual stratum on the basis of the "relative" stratum size, N, and the stratum standard deviation, SD.

$$n_i = n \times N_iSD_i / \Sigma N_iSD_i \qquad \text{(eqn 3.2)}$$

In equation 3.2, SD_i represents the standard deviation of stratum i. All other variables are the same as in equation 3.1.

Although the optimal allocation method is generally more effective, the proportional allocation method will be utilized in this study. Proportional allocation will give more meaningful results (for comparison with the Basket Method) for this investigation given that strata boundaries are being set using the "equal dollar value per strata rule." If optimal allocation were used, in two strata the sample sizes calculated using equation 3.2 would be greater than the total number of elements in the strata. If this were to happen in a real world sampling situation, each affected strata sample size would be set equal to its population size and sample sizes would be recalculated for the remaining strata. The saturated strata would then be audited 100 percent. To do this for this study would not facilitate comparison of the two methods of sampling under "like" circumstances.

The sample size computations depend on whether the optimal or proportional allocation method is used. There are equations to be used for each method in calculating the appropriate sample size required to achieve a stated level of precision and reliability. The equations will not be enumerated here because sample size requirement calculations are not required to be made for the purposes of this study. This is because in actual applications where the true value of the population is not known the only way to be reasonably certain that one's results are valid is by complying with rules which will tie the audit to statistical theory. The sampling rules for Stratified Random Sampling are designed to do just that, so that the auditor who follows the sampling procedures will be able to determine the extent of the audit required to achieve the desired level of certainty.

In this study the true values of the proposals are known, as are the size and distribution of errors, and the Stratified Random Sampling method is not being compared to its theoretical limits, but to a second method to determine which of the two is the more desirable in a certain case. The sample size is this study will be chosen arbitrarily, and is further described in the Description of Simulation section.

4. **Select a Random Sample of Size n_i From the Strata**
5. **Calculate the Mean of Each Stratum Based On n_i for each Stratum**
6. **Calculate the Estimated Audited Population Total**

This calculation involves taking the mean of each stratum (derived in step 5 above), multiplying it by the total number of items in the stratum and then summing the results. This gives the *estimated audited population* total which can be mathematically represented as follows:

$$\Sigma\ \bar{x}_i N_i \qquad \text{(eqn 3.3)}$$

where $\bar{x}_i$ represents the mean of stratum i, N_i, the total number of items within stratum i, and $\Sigma\ \bar{x}_i N_i$ the sum of ($\bar{x}_i N_i$).

7. **Check Reliability of the Estimated Audited Population Total**

This step involves concluding that one is certain at the reliability specified in Step 1 that the true book value is within the *estimated audited population* total plus-or-minus the **achieved** precision.[6]

---

[6] There is a formula which can be used to calculate the achieved precision. The achieved precision should always be less than or equal to the desired or acceptable precision. If the achieved precision is greater than the acceptable precision, the sample size is unsufficient because the precision limit is too wide. If this were the case the sample size would have to be increased.

IV. DESCRIPTION OF SIMULATION

A. DERIVING COMPARABLE RESULTS

As mentioned previously in this paper, the desired sampling technique is not necessarily the one that results in the most accurate, average estimate for proposal populations with varying error arrangements. A more important characteristic of the desired method will be that it responds least to variations in the arrangement of errors. In other words, it will be the method which is more consistent in its predictions over various error arrangements and patterns. Since it is the consistency of the drawn sample which is of interest in this investigation, the sample selection process of both the Basket Method and the Stratified Random Sampling Method will be used to draw samples from the same populations. The samples will then be evaluated according to the basket method, which will give an estimate of the true value of the population, to see how well each method's sample reflected the value of the population.

The rules of the basket method will create the identical set of baskets from a given population every time they are applied. Therefore, the "baskets" of the basket method can easily be evaluated by a complete review of the specific results. The rules of the Stratified Random Sampling technique also provide a finite number of samples, but that number is significantly greater than the number of different baskets.

To evaluate the sample drawn with the Stratified Random Sampling rules, the sample is treated as if it were a basket. Then, the sample is evaluated using the basket method evaluation technique; that is, all of the sample's resident proposals are audited and their true value is divided by their proposal value. The resulting factor is multiplied against the population proposal total to determine the best estimate of the true total value of the proposal population.

B. CREATION OF THE TEST POPULATIONS

Using the general purpose statistical computing system Minitab, the original population was seeded with errors at a 5% and 10% rate of occurrence in a random distribution. The 5% error population (population A) was then skewed to form two additional test populations. One (population B) had its errors skewed strongly to its higher valued proposals, and the other (population C) to its lower valued proposals. The total dollar amount of error and number of overstated proposals remained constant during the skewing procedure. All populations were created in both a "dishonest" version which had overstatements only and are named with single letters (populations A,B,C,D, and E), and in an "honest" version with both overstatements and understatements named by double letters (AA,BB,CC,DD, and EE). Except for the sign on each error, the single letter named populations are identical to their double letter named counterparts. Therefore, populations AA, BB, and CC differ from their single lettered counterparts only in the fact that they contain both errors of overstatement and understatement. The populations with 10% errors differ in that E and EE, while containing the same number of errors in the same distribution and sign as D and DD respectively, have errors of much larger magnitude, so that the sum of the dollar value of the errors make up 10% of the population in E and EE but only 1% of the population in D and DD. The populations are described in Table 1.

TABLE 1

POPULATION DESCRIPTION

NAME	A	B	C	D	E
Population	8,300	8,300	8,300	8,300	8,300
Erroneous Proposals	5%	5%	5%	10%	10%
Σ $ Errors/Σ $ Proposals	9%	9%	9%	1%	10%
Types of Errors	+	+	+	+	+
Skew (none, high, or low)	N	H	L	N	N

NAME	AA	BB	CC	DD	EE
Population	8,300	8,300	8,300	8,300	8,300
Erroneous Proposals	5%	5%	5%	10%	10%
Σ $ Errors/Σ $ Proposals	9%	9%	9%	1%	10%
Types of Errors	+/-	+/-	+/-	+/-	+/-
Skew (none, high, or low)	N	H	L	N	N

C. SIMULATION EXECUTION

Simulations were run on all populations using both a Basket Method evaluating program and a Stratified Random Sampling procedure which was done manually within Minitab. The Basket Method program utilized was written by Lieutenant James

P. Tortorelli for use in his thesis at the Naval Postgraduate School [Ref. 2: pg. 18]. This program, written in Waterloo BASIC, is listed in Appendix A. The Stratified Random Sampling simulation process is detailed in Appendix B and consists of the following basic steps. Each step is referenced by line number to its actual application in Appendix B:

1. **Stratify the Population**
   (Appendix B line numbers 6 - 45)
2. **Allocate the Total Sample to the Strata**
   (Line numbers 48 - 80)
3. **Select a Random Sample From Each Strata**
   (Line numbers 81 - 130)
4. **Calculate the "Book Value" sum for Audited Items**
   (Line numbers 131 - 133)
5. **Calculate the "Audit Value" sum for Audited Items**
   (Line numbers 134 - 136)
6. **Calculate the Correction Factor**
   (Line numbers 137 - 139)
7. **Calculate the Predicted Population Audit Total**
   (Line numbers 140 - 142)
8. **Calculate the Percent Error**
   (Line numbers 143 - 148)

As mentioned earlier, ten baskets were arbitrarily chosen for the Basket Method; this resulted in 830 proposals per basket. Ten strata were then chosen for the Stratified Random Sampling Method with a total sample size of 830 to be selected. Strata boundaries were set using the "equal dollar value per strata" rule; therefore, each strata has approximately the same total dollar amount contained within it. This rule was used because it allows better comparison with the Basket Method in that each "basket" formed under the Basket Method sample selection process has nearly equal dollar basket totals. Ten trials were run using the Stratified Random Sampling selection method.

The ten audit results for each sample selected by the two methods were then divided by their respective proposal sums to derive the correction factors as follows.

$$F = \text{Total audit sum of sample/Total proposal sum of sample} \qquad \text{(eqn 4.1)}$$

These correction factors were multiplied by the sum of all proposals to derive the predicted true audit total for the population. That is,

$$PTAT = F \times PSUM \qquad \text{(eqn 4.2)}$$

where PTAT represents the predicted true audit total for the population, F, the correction factor, and PSUM the sum of all proposals. The difference between the predicted true audit total and the actual audit total was then divided by the sum of all proposals to give a percent error for each basket and trial. This calculation can be mathematically represented as follows:

$$PE = (PTAT - AAT)/PSUM \qquad \text{(eqn 4.3)}$$

where PE represents percent error, PTAT, the predicted true audit total for the population, AAT, the actual audit total for the population, and PSUM the sum of all proposals. The mean percent error for each method was then calculated in the following manner:

$$MPE = \Sigma PE/10 \qquad \text{(eqn 4.4)}$$

where MPE represents the mean percent error and ΣPE the sum of the individual percent error amounts for each trial. The mean percent errors for each method by population are listed in Table 2.

TABLE 2
SIMULATION RESULTS (MEAN % ERROR)

NAME	A	B	C	D	E
Basket Method	.770	.571	.725	.089	.629
SRS	-.898	1.011	.783	-.078	-.659

NAME	AA	BB	CC	DD	EE
Basket Method	.901	.853	1.099	.065	.774
SRS	-.936	-1.253	1.367	-.105	.971

Detailed results are shown in Appendix C. A positive percent error represents an overestimate and a negative percent error represents an underestimate. With the exception of population D, the Basket Method of sample selection was always more accurate with overstatement errors. For the populations with both overstatement and understatement errors, the Basket Method was more accurate across the board. The data from Table 2 are perhaps more vividly illustrated when expressed in a different manner. The Basket Method errors are expressed as a percent of the Stratified Random Sampling errors in Table 3.

TABLE 3

BASKET METHOD ERROR AS A % OF STRATIFIED RANDOM SAMPLING ERROR

NAME	A	B	C	D	E
Percent	85	56	92	114	95
NAME	AA	BB	CC	DD	EE
Percent	96	68	80	61	79

The average or mean value (in this case mean percent error) in a set of measurements is only one important summary figure. It is also important to summarize the extent to which values differ among themselves or about a central value. One of the most useful statistical measures of variability is the standard deviation. This measure is based on the concept of deviations from the mean. The deviation of a sample measurement y_i from its mean $\bar{y}$ is defined as $(y_i - \bar{y})$. The standard deviation of a sample of "n" measurements $y_1, y_2, \ldots, y_n$ is defined to be the square root of the sum of the squared deviations divided by (n - 1). The standard deviation, s, can be denoted as follows.

$$s = \sqrt{\Sigma(y_i - \bar{y})^2 / n - 1} \qquad \text{(eqn 4.5)}$$

As previously mentioned, the measure of standard deviation may be used to show the degree of variation among values in a given set of data, or it may be used to supplement an average to describe a group of data. It also may be used to compare one group of data with another. When the standard deviation is high, the average (mean) is of less significance as a statistical measure. When the standard deviation is low, the value of the average is considered to be a highly representative value.

The standard deviations of the percent error for each method were calculated using the data from Appendix C. The results are given in Table 4.

TABLE 4

STANDARD DEVIATION OF MEAN PERCENT ERROR

NAME	A	B	C	D	E
Basket Method	.725	.468	.473	.083	.462
SRS	.698	.584	.381	.059	.470

NAME	AA	BB	CC	DD	EE
Basket Method	.530	.796	.800	.055	.601
SRS	.915	1.028	1.332	.062	.586

In looking at the results in Table 4, the significance of the standard deviation figures lies not so much in whether they are considered to be high or low; the significance lies in the comparable sizes of the standard deviations between the Basket Method and the Stratified Random Sampling Method. What this means is that the mean percent errors for both methods have about the same "representativeness" as far as being a good summary statistic. This lends more credibility to the simulation results as a basis for comparison of the two methods.

V. SUMMARY AND CONCLUSIONS

A. RESISTANCE TO PROPOSAL RIGGING

In order to benefit from the potential time and labor savings a sampling system offers, the sampling technique must be resistant to padding schemes. If not, a dishonest contractor has much to gain by trying to selectively pad proposals. Therefore, as mentioned previously, the primary goal is not necessarily to determine which of the two methods is the most accurate, but to see which one least benefits attempted padding schemes. When comparing a method's performance between the single and double letter versions of a population it can be seen in Table 2 that both the Basket Method (with the exception of population D) and the Stratified Random Sampling method are stricter when estimating the value of the overstatement-only population than when estimating the value of the "honest" populations. Therefore, padding one's contract proposals with overstatements in random, low, or high skewed distributions prior to submitting them to either method for evaluation is not likely to raise the resulting estimate for the population, but is instead likely to lower the estimated value. However, except for population D, the samples drawn with Stratified Random Sampling allowed the overstatement only (padded) populations a larger estimate than did the sample drawn with the Basket Method.

B. EVALUATION

Assuming honest contractors are as likely to understate as overstate their costs and dishonest contractors are not, honest contractors will be more successful (except for populations D and DD under the Basket Method) than dishonest contractors under either of the sampling methods. Since the Basket Method allows less benefit to accrue to the dishonest contractor than Stratified Random Sampling, and because it gives a more accurate estimate in general, the Basket Method is judged to be a more desirable sampling method for the purposes addressed in this paper.

C. AREAS FOR FURTHER RESEARCH

Some suggestions for further study are:

1. Fewer or more baskets and strata may be used.
2. The Basket Method can be compared to other sampling methods.
3. A data set with much smaller variance in proposal size could be used.
4. Additional error arrangement strategies can be developed and tested.

APPENDIX A

BASKET METHOD PROGRAM LISTING

```
00100 REM THIS IS A PROGRAM TO PROCESS DATA USING THE BASKET METHO
00120 REM DATA IS INPUT FROM A FILE, SEPARATED BY COMMAS, AND LISTE
00140 REM AS PAIRS OF VALUES FOR A BID, THE BOOK FIRST AND THE
00160 REM AUDITED VALUE SECOND.  THE PROGRAM EXPECTS 'DATPOP' PAIR
00180 REM OF VALUES.  DATA MUST BE IN DESCENDING ORDER BY BOOK VAL
00200 REM
00220 REM  **  DIMENSION VARIABLES  **
00240 REM
00260 DIM ASUM(50), BSUM(50), ANEXT(50), BNEXT(50)
00280 DIM ERRORP(50), FACTOR(50), ERRORA(50)
00300 REM
00320 REM  **  SET CONSTANTS  **
00340 REM
00360 B = 10             ! NUMBER OF BASKETS
00380 DATPOP = 8300      ! NUMBER OF DATA PAIRS
00400 BPOP=INT(DATPOP/B) ! INITIATE RUNNING TALLY OF DATA PAIRS READ
00420  OPEN #3, 'TEST (RECFM F LRECL 80)', INPUT
00440 ATOT = 0
00460 BTOT = 0
00480 BPOP1 = 1
00500 FOR J = 1 TO 10
00520  ASUM(J) = 0
00540  BSUM(J) = 0
00560  NEXT J
00580 EES = 0             !  SUM OF ERROR SQUARES
00600 EED = 0             !  SUM OF BASKET DOLLAR SQUARES
01000  REM
01020  REM **  ROUTINE TO READ IN DATA  **
01040  REM
01060  IF BPOP1 > BPOP
01080    GOTO 4000      ! IF NO MORE DATA, THEN PROCESS
01100    ENDIF
01120  FOR I = 1 TO B
01140    INPUT #3, BNEXT(I), ANEXT(I)
01160    NEXT I
01180  BPOP1 = BPOP1 + 1
02000 REM
02020 REM  **  ROUTINE TO SORT PARTIAL SUMS IN  **
02040 REM  **   BASKETS IN ASCENDING ORDER     **
02060 REM
02080  I = 1
```

```
02100 WHILE I < B
02120   IF BSUM(I) > BSUM(I+1)
02140     C1 = BSUM(I)
02160     C2 = ASUM(I)
02180     BSUM(I) = BSUM(I+1)
02200     ASUM(I) = ASUM(I+1)
02220     BSUM(I+1) = C1
02240     ASUM(I+1) = C2
02260     IF I > 1
02280       I = I-1
02300       ENDIF
02320     GOTO 2120
02340     ENDIF
02360   I = I+1
02380   ENDLOOP
03000 REM
03020 REM  **  ADD NEXT ROUND TO BASKETS  **
03040 REM
03060 FOR I = 1 TO B
03080   BSUM(I) = BSUM(I) + BNEXT(I)
03100   ASUM(I) = ASUM(I) + ANEXT(I)
03120   NEXT I
03140 GOTO 1060
04000 REM
04020 REM  **  ADDING ROUTINE  **
04040 REM
04060 FOR I = 1 TO B
04080   BTOT = BTOT + BSUM(I)
04100   ATOT = ATOT + ASUM(I)
04120   NEXT I
04140 FOR I = 1 TO B
04160   FACTOR(I) = ASUM(I)/BSUM(I)
04180   ERRORA(I) = BTOT * FACTOR(I) - ATOT
04185   EES = EES + ERRORA(I) * ERRORA(I)
04190   EED = EED + BSUM(I) * BSUM(I)
04200   ERRORP(I) = 100 * ERRORA(I)/BTOT
04220   MAE = MAE + ABS(ERRORA(I))
04240   MPE = MPE + ABS(ERRORP(I))
04260   NEXT I
04280 MAE = MAE / B
04300 MPE = MPE / B
05000 REM
05020 REM  **  PRINT RESULTS  **
05040 REM
05060 PRINT 'BASKET  BOOK VALUE  AUDIT VALUE  FACTOR  %ERROR  ERR OF F
05080 FORM0S='TOTAL #######.##   #######.##   #.#### ##.####  #####.##'
05100 FORM1S=' ##   #######.##   #######.##   #.####  #.####  #####.##'
```

```
05120 FORM2S= ' MEAN                              #.####  #####.##'
05140 PRINT USING FORM0S, BTOT, ATOT, ATOT/BTOT, 100*(BTOT-ATOT)/BTOT,
05160 &      BTOT - ATOT
05180 FOR I = 1 TO B
05200   PRINT USING FORM1S, I, BSUM(I), ASUM(I), FACTOR(I), ERRORP(I),&
05220 &      ERRORA(I)
05240   NEXT I
05260 PRINT USING FORM2S, MPE, MAE
05280 FORM3S= '  DOLLARS     #######.##     #######.##'
05300 FORM4S= 'CONTRACTS     #######.##     #######.##'
06000 PRINT '             MEAN AUDITED       S. D.'
06020 PRINT USING FORM3S,BTOT/B,SQR(((B*EED)-(BTOT*BTOT))/(B*(B-1)))
06040 PRINT USING FORM4S,DATPOP/B,0
06060 REM
06080 REM  **  CLEANUP  **
06100 REM
06120 CLOSE #3
07000 END
```

APPENDIX B

MINITAB SIMULATION OF STRATIFIED RANDOM SAMPLING

```
MTB > name c22 'book' c50 'audit'
MTB > sum c22 k1
  SUM    =     409605
MTB > sum c50 k2
  SUM    =     372255
MTB > copy c22 c50 c51 c52;
SUBC> use 'book' = .50:18.75.
MTB > count c51 k3
  COUNT   =      3328.0
MTB > copy c22 c50 c53 c54;
SUBC> use 'book' = 18.76:27.21.
MTB > count c53 k4
  COUNT   =      1807.0
MTB > copy c22 c50 c55 c56;
SUBC> use 'book' = 27.22:43.50.
MTB > count c55 k5
  COUNT   =      1228.0
MTB > copy c22 c50 c57 c58;
SUBC> use 'book' = 43.51:79.00.
MTB > count c57 k6
  COUNT   =      698.00
MTB > copy c22 c50 c59 c60;
SUBC> use 'book' = 79.01:107.35.
MTB > count c59 k7
  COUNT   =      429.00
MTB > copy c22 c50 c61 c62;
SUBC> use 'book' = 107.36:141.36.
MTB > count c61 k8
  COUNT   =      341.00
MTB > copy c22 c50 c63 c64;
SUBC> use 'book' = 141.37:219.82.
MTB > count c63 k9
  COUNT   =      238.00
MTB > copy c22 c50 c65 c66;
SUBC> use 'book' = 219.83:436.65.
MTB > count c65 k10
  COUNT   =      135.00
MTB > copy c22 c50 c67 c68;
SUBC> use 'book' = 436.66:906.31.
MTB > count c67 k11
  COUNT   =      67.000
```

```
42) MTB > copy c22 c50 c69 c70;
43) SUBC> use 'book' = 906.32:2440.00.
44) MTB > count c69 k12
45)   COUNT   =     29.000
46) MTB > let k13 = 830
47) MTB > let k14 = 8300
48) MTB > let k15 = k14 * (k3/k14)
49) MTB > round k15 k15
50)    ANSWER =     3328.0000
51) MTB > let k15 = k13 * (k3/k14)
52) MTB > round k15 k15
53)    ANSWER =      333.0000
54) MTB > let k16 = k13 * (k4/k14)
55) MTB > round k16 k16
56)    ANSWER =      181.0000
57) MTB > let k17 = k13 * (k5/k14)
58) MTB > round k17 k17
59)    ANSWER =      123.0000
60) MTB > let k18 = k13 * (k6/k14)
61) MTB > round k18 k18
62)    ANSWER =       70.0000
63) MTB > let k19 = k13 * (k7/k14)
64) MTB > round k19 k19
65)    ANSWER =       43.0000
66) MTB > let k20 = k13 * (k8/k14)
67) MTB > round k20 k20
68)    ANSWER =       34.0000
69) MTB > let k21 = k13 * (k9/k14)
70) MTB > round k21 k21
71)    ANSWER =       24.0000
72) MTB > let k22 = k13 * (k10/k14)
73) MTB > round k22 k22
74)    ANSWER =       13.0000
75) MTB > let k23 = k13 * (k11/k14)
76) MTB > round k23 k23
77)    ANSWER =        7.0000
78) MTB > let k24 = k13 * (k12/k14)
79) MTB > round k24 k24
80)    ANSWER =        3.0000
81) MTB > sample k15 c51 c52 c71 c72
82) MTB > sum c71 k25
83)   SUM     =     4025.3
84) MTB > sum c72 k26
85)   SUM     =     4025.3
86) MTB > sample k16 c53 c54 c73 c74
87) MTB > sum c73 k27
88)   SUM     =     4135.1
```

```
 89) MTB >  sum c74 k28
 90)    SUM      =       4135.1
 91) MTB >  sample k17 c55 c56 c75 c76
 92) MTB >  sum c75 k29
 93)    SUM      =       4099.2
 94) MTB >  sum c76 k30
 95)    SUM      =       4099.2
 96) MTB >  sample k18 c57 c58 c77 c78
 97) MTB >  sum c77 k31
 98)    SUM      =       4081.9
 99) MTB >  sum c78 k32
100)    SUM      =       4081.9
101) MTB >  sample k19 c59 c60 c79 c80
102) MTB >  sum c79 k33
103)    SUM      =       4127.1
104) MTB >  sum c80 k34
105)    SUM      =       2597.1
106) MTB >  sample k20 c61 c62 c81 c82
107) MTB >  sum c81 k35
108)    SUM      =       4048.2
109) MTB >  sum c82 k36
110)    SUM      =       2608.2
111) MTB >  sample k21 c63 c64 c83 c84
112) MTB >  sum c83 k37
113)    SUM      =       4264.7
114) MTB >  sum c84 k38
115)    SUM      =       3994.7
116) MTB >  sample k22 c65 c66 c85 c86
117) MTB >  sum c85 k39
118)    SUM      =       3937.9
119) MTB >  sum c86 k40
120)    SUM      =       3757.9
121) MTB >  sample k23 c67 c68 c87 c88
122) MTB >  sum c87 k41
123)    SUM      =       4700.3
124) MTB >  sum c88 k42
125)    SUM      =       4520.3
126) MTB >  sample k24 c69 c70 c89 c90
127) MTB >  sum c89 k43
128)    SUM      =       3892.9
129) MTB >  sum c90 k44
130)    SUM      =       3892.9
131) MTB >  let k45 = k25+k27+k29+k31+k33+k35+k37+k39+k41+k43
132) MTB >  prin k45
133) K45      41312.6
134) MTB >  let k46 = k26+k28+k30+k32+k34+k36+k38+k40+k42+k44
135) MTB >  prin k46
```

```
K46      37712.6
MTB >  let k47  =  k46/k45
MTB >  prin k47
K47      0.912859
MTB >  let k48  =  k47 * k1
MTB >  prin k48
K48      373912
MTB >  let k49  =  (k48 - k2)/k1
MTB >  prin k49
K49      0.00404459
MTB >  let k50  =  k49 * 100
MTB >  prin k50
K50      0.404459
```

APPENDIX C
DETAILED RESULTS

POPULATION A

Total Book Value: $409,605.73
Total Audit Value: $372,255.73

Trial	Percent Error Using Basket Method	Percent Error Using SRS
1	-1.211	.404
2	2.307	.609
3	.769	-.057
4	.330	.948
5	.330	-2.429
6	-1.651	.375
7	-.330	-1.488
8	.110	-1.397
9	-.330	.609
10	-.330	-.663
Mean	.770	-.898

POPULATION AA

Total Book Value: $409,605.73
Total Audit Value: $410,055.73

Trial	Percent Error Using Basket Method	Percent Error Using SRS
1	.110	-.110
2	-.769	-.325
3	1.648	-.109
4	-.989	-1.843
5	1.648	-1.644
6	-1.210	.562
7	-1.210	-1.838
8	-.330	2.501
9	.549	.106
10	.549	-.326
Mean	.901	-.936

POPULATION B

Total Book Value: $409,515.73
Total Audit Value: $372,255.73

Percent Error Using Percent Error Using

Trial	Basket Method	SRS
1	1.187	.540
2	-1.231	-1.295
3	.088	.447
4	.308	2.016
5	1.187	-.540
6	.088	.103
7	-.132	1.403
8	-.352	1.106
9	-.571	-1.440
10	-.571	-1.217
Mean	.571	1.011

POPULATION BB

Total Book Value: $409,515.73
Total Audit Value: $408,975,73

Trial	Percent Error Using Basket Method	Percent Error Using SRS
1	1.890	-.092
2	.352	-3.139
3	-.527	-1.682
4	.132	-.085
5	-.747	1.042
6	-.088	.572
7	-2.068	-1.415
8	-.088	1.663
9	1.890	2.492
10	-.747	.346
Mean	.853	-1.253

POPULATION C

Total Book Value: $409,605.73
Total Audit Value: $372,255.73

Trial	Percent Error Using Basket Method	Percent Error Using SRS
1	.769	-.173
2	.549	.565
3	-.110	-.706
4	.110	-1.056
5	.549	1.182
6	-.549	1.250
7	-.769	-1.232
8	-.989	.747
9	-1.211	.367
10	1.648	-.557

Mean	.725	.783

POPULATION CC

Total Book Value: $409,605.73
Total Audit Value: $409,155.73

Trial	Percent Error Using Basket Method	Percent Error Using SRS
1	-1.210	.994
2	2.527	4.715
3	-.769	-1.679
4	-2.310	-.323
5	-.110	-.555
6	.110	-.994
7	-.549	-1.245
8	1.428	.329
9	-.549	2.297
10	1.428	.536
Mean	1.099	1.367

POPULATION D

Total Book Value: $375,991.73
Total Audit Value: $372,256.73

Trial	Percent Error Using Basket Method	Percent Error Using SRS
1	.108	-.201
2	.003	-.096
3	-.024	-.031
4	-.096	.037
5	-.048	.117
6	-.132	.127
7	.287	.039
8	-.024	-.015
9	.048	-.033
10	-.119	-.087
Mean	.089	-.078

POPULATION DD

Total Book Value: $375,991.73
Total Audit Value: $376,360.73

Trial	Percent Error Using Basket Method	Percent Error Using SRS
1	-.026	.048
2	-.086	-.170
3	.057	-.013
4	.033	-.133
5	-.134	-.015

6	-.002	-.156
7	-.014	-.170
8	.059	-.145
9	-.062	.076
10	.177	.126
Mean	.065	-.105

POPULATION E

Total Book Value: $413,756.73
Total Audit Value: $372,256.73

Trial	Percent Error Using Basket Method	Percent Error Using SRS
1	-.483	.146
2	-.346	.103
3	.967	-.944
4	-.362	-.404
5	-1.090	.434
6	.725	-.231
7	1.450	-.601
8	-.121	1.170
9	-.121	-1.279
10	-.121	-1.279
Mean	.629	-.659

POPULATION EE

Total Book Value: $413,756.73
Total Audit Value: $417,856.73

Trial	Percent Error Using Basket Method	Percent Error Using SRS
1	.338	1.267
2	-1.961	-.375
3	.822	1.001
4	-.024	1.295
5	.943	1.295
6	1.305	-1.946
7	-1.111	.105
8	-.024	-1.471
9	-.749	.459
10	.459	.500
Mean	.774	.971

LIST OF REFERENCES

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